

Dr Zlatko Kovacic (1995) *Time series analysis*. Faculty of Economics, University of Belgrade (set text for the fourth year course in *Time series analysis*).

TIME SERIES ANALYSIS

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